What are Logarithms?

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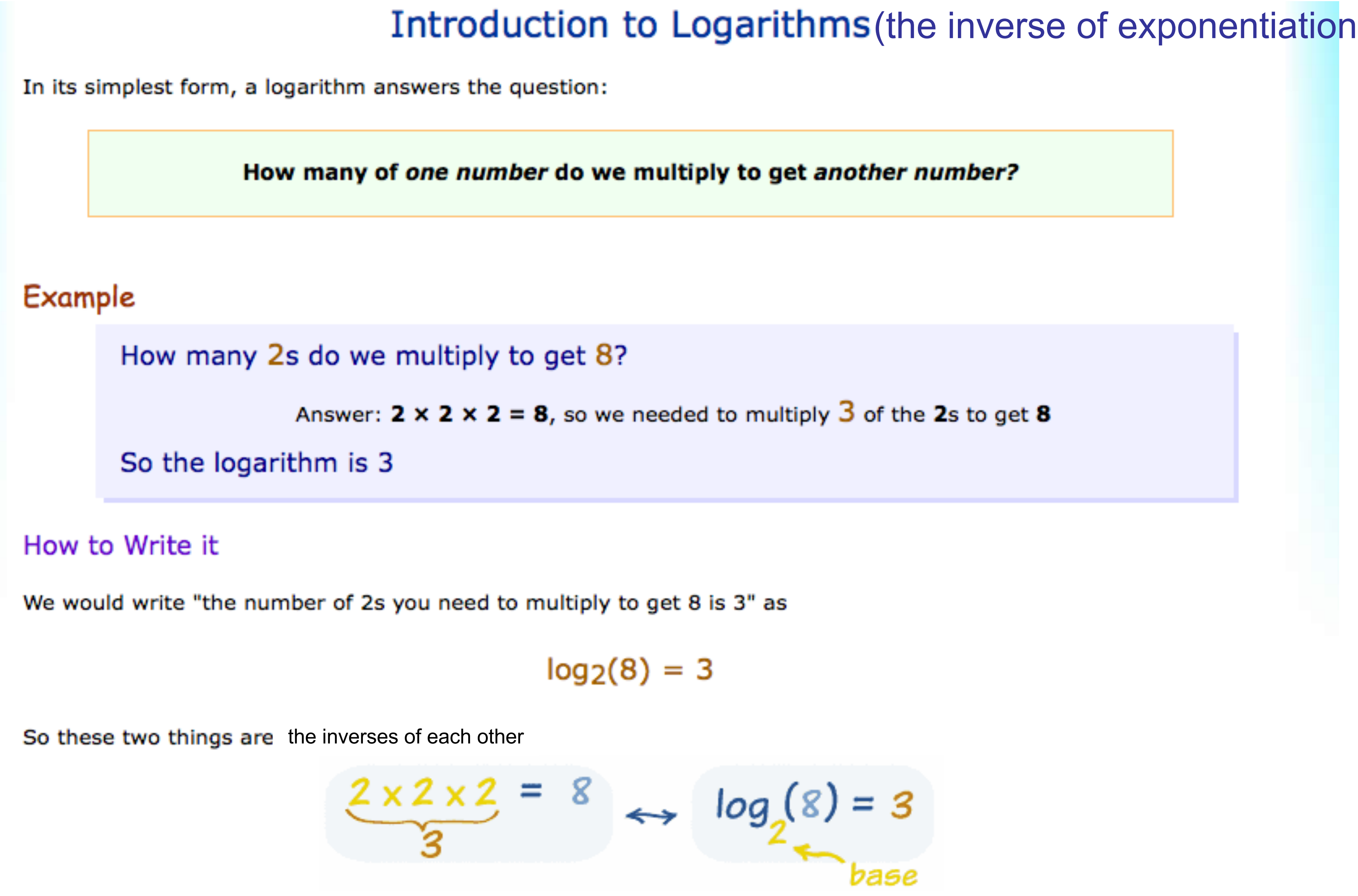
Logarithm = *Exponent of the Base*



Introduction to Logarithms(the inverse of exponentiation)

In its simplest form, a logarithm answers the question:

How many of *one number* do we multiply to get *another number?*

Example

How many 2s do we multiply to get 8?

Answer: **2 × 2 × 2 = 8**, so we needed to multiply 3 of the **2**s to get **8**

So the logarithm is 3

How to Write it

We would write "the number of 2s you need to multiply to get 8 is 3" as

$$\log_2(8) = 3$$

So these two things are the inverses of each other

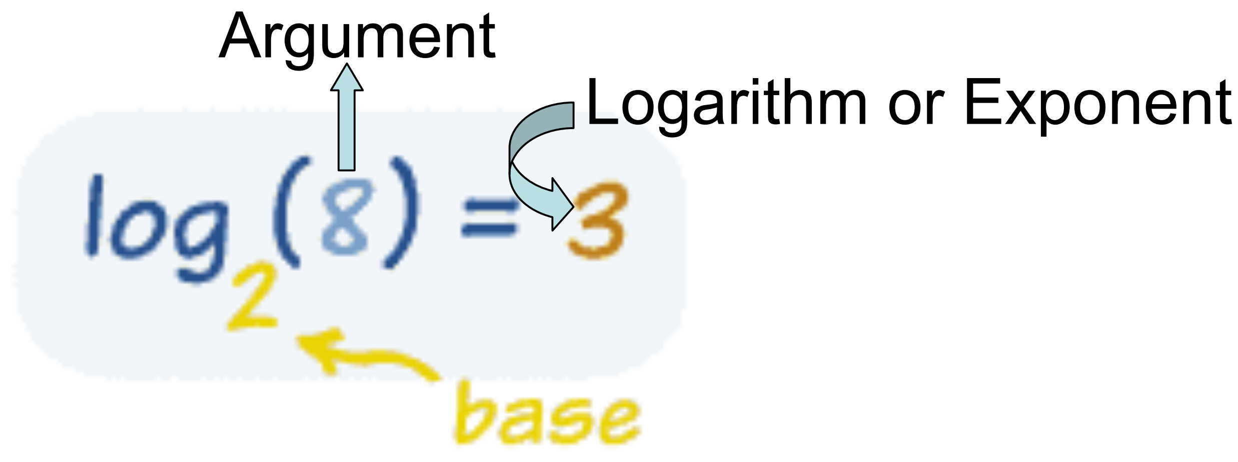
$$\underbrace{2 \times 2 \times 2}_{3} = 8 \quad \leftrightarrow \quad \log_2(8) = 3$$

base

Logarithm Nomenclature:
The 3 Numbers

Notice we are dealing with three numbers:

- the **base**: the number we are multiplying (a "2" in the example above)
- how many times to use it in a multiplication (3 times, which is the **logarithm**)
- The number we want to get (an "8") *Called the* "**Argument**"



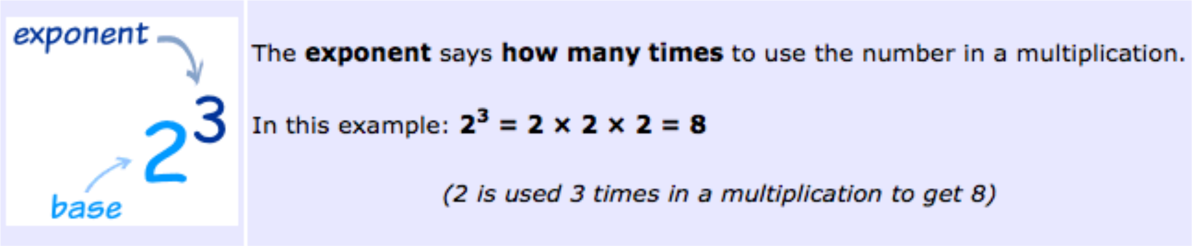
Argument

Logarithm or Exponent

Logarithm = *Exponent of the Base*

Exponents

The exponent of a base = a Logarithm   Let’s see how:



The **exponent** says **how many times** to use the number in a multiplication.

In this example: $\mathbf{2^3 = 2 \times 2 \times 2 = 8}$

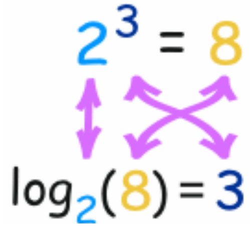
(2 is used 3 times in a multiplication to get 8)

So when you have a question like this:

$$2^{?} = 8$$

What is the exponent of 2 that equals 8?

The Logarithm answers it like this:



$$2^3 = 8$$

$$\log_2(8) = 3$$

The logarithm tells you what the exponent is!!!

It is read: “The log base 2 of 8 equals 3 and 3 is the: ***<u>EXPONENT OF THE BASE</u>***”

Summary

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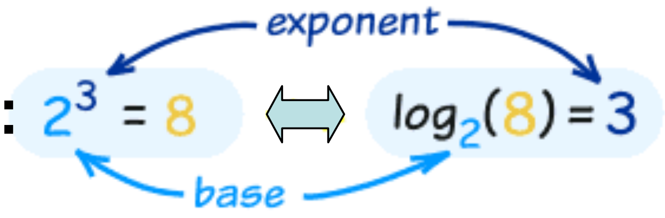
In this case the base is 2, the argument is 8 and the exponent [$\log_2(8)$] is 3

The inverse of log:



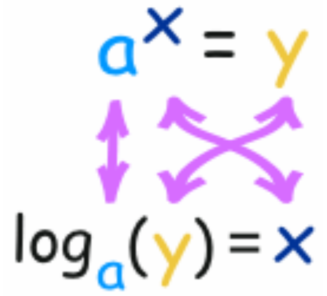
$2^3 = 8 \Leftrightarrow \log_2(8) = 3$

The inverse of exponentiation

So the logarithm answers the question:

What exponent do we need
(for one number to become another number) ?

The General Case is:



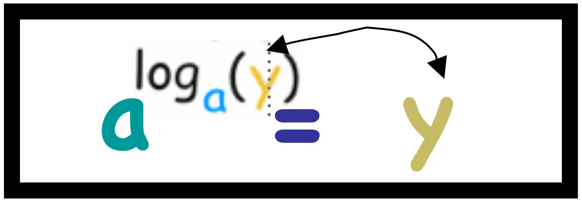
$a^x = y$

$\log_a(y) = x$

or base$^{\text{exponent}}$ = argument

or $\log_{\text{base}}$(argument) = exponent

Because x and $\log_a(y)$ are equal we can write:



$a^{\log_a(y)} = y$

Solving an Equation that Contains a Log on One Side

If: $2\log_a(y) = x$, solve this equation for y

Raise the base of the log
to the power (exponent) that is
equal to an entire side of the above equation, if and only if
one side of the equation contains only the log.

In this case the base of the log is "a" and the log is NOT
the only mathematical expression on the left hand
side of the equation since there is a 2 in front of the log. Therefore we can't raise "a" to
each side of the above equation until we divide both sides by 2 as follows:

$$\log_a(y) = x/2$$

$$a^{\log_a(y)} = a^{x/2}$$

So, $\boxed{y = a^{x/2}}$

But we now know that:

$$a^{\log_a(y)} = y$$

The Most Common Bases of Logarithms: 10 and e

"The Common Logarithm": Base 10

Sometimes you'll see a logarithm written without a base, like this: $\log(100)$

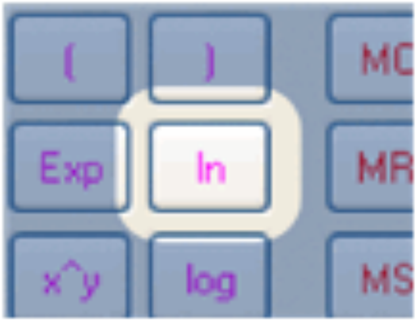
This usually means that the base is really 10: $\log_{10}(100)$
The base 10 logarithms are called "Common Logarithms"

Electrical Engineers love to use this base. On a calculator it is the "log" button and some calculators will have $\log_{10}$ on the button:

The Most Common Bases of Logarithms: 10 and e

- “The Natural Logarithm”: Base “e”
- “e” is a special number (sometimes called Euler’s number) and it is the base of the “natural logarithm”. It is a constant number and its value is: 2.71828……..
- It is commonly used in Math, Science and Engineering
- In computer science: e is written as exp with the exponent inside parentheses

  For example $e^2 = \exp(2)$ or $e^x = \exp(x)$
- The logarithm of y to the base of e is written as: ln(y) and is not written as $\log_e(y)$, although that would be correct.
- The logarithm base “e” is the exponent of “e”, so that it is also the number of times you need to use “e” in a multiplication to get the “Argument”.
- On a calculator it is the “ln” button:



Changing to Known (on a calculator) Bases

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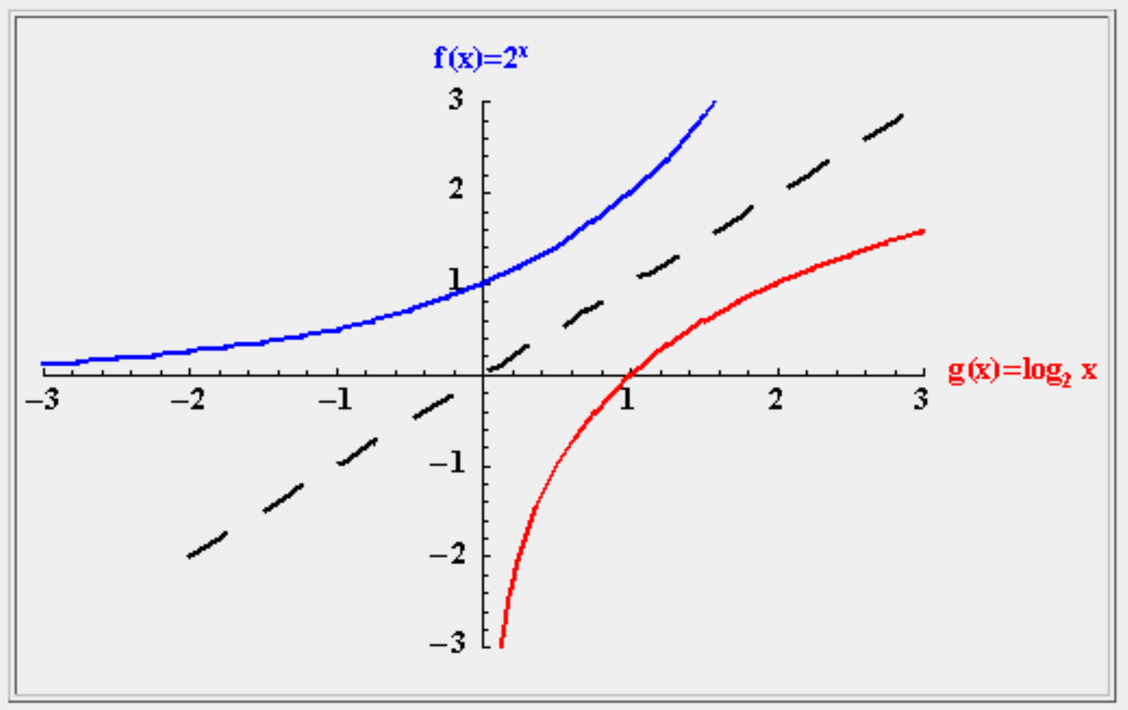
$$\log_b x = \frac{\log_k x}{\log_k b}$$

$$\log_6 15 = \frac{\ln 15}{\ln 6} = \frac{\log 15}{\log 6} = 1.51139$$

Logarithms and Logarithm Properties

Logarithmic functions are the inverse of exponential functions. For example if (4, 16) is a point on the graph of an exponential function, then (16, 4) would be the corresponding point on the graph of the inverse logarithmic function.



Finding the Inverse

If $y=\log_b x$: Make each side the exponent of the base, switch variables x and y and re-solve for y

eg: $y = \log_2 x$

$2^y = 2^{\log_2 x}$ Raise base to each side

$2^x = 2^{\log_2 y}$ Switch x and y

$2^x = y$ Solve for y

If $y=b^x$: Isolate b^x, take the log base b of both sides, switch variables x and y and re-solve for y

eg: $y = 2^x$ 2^x already isolated

$\log_2 y = x\log_2 2 = x$ Take log base 2 of both sides

$\log_2 x = y$ Switch x and y

$y = \log_2 x$ Solve for y

Logarithm Properties

The domain of the logarithm function is: $(0,\infty)$
In other words, we can only plug positive numbers into a logarithm! We can't plug in zero or a negative number.

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

Notice that these last 2 properties tell us that:

$f(x) = b^x$ and $g(x) = \log_b x$

are INVERSES of each other

Logarithms Have Exponent Properties

Since Logarithms are exponents, they have the same Properties of Exponents:

$$\log_b(x^r) = r\log_b x$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(\frac{x}{y}) = \log_b x - \log_b y$$

Special Logarithms:

Common Logarithm: $\log x = \log_{10} x$

Natural Logarithm: $\ln x = \log_e x$

Where e = 2.718281828…